K → ππ at physical point with periodic BC

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$K \to \pi\pi$ & CP violation

CP odd CP even

$$|K_L\rangle = |K_2\rangle + \varepsilon |K_1\rangle$$

ε' direct CPV

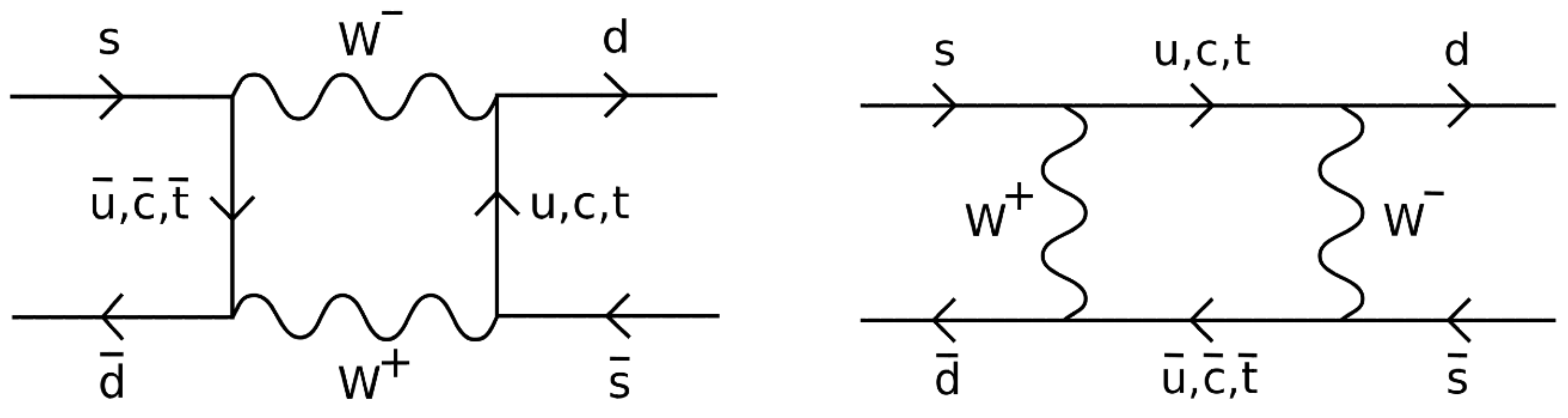
ε indirect CPV

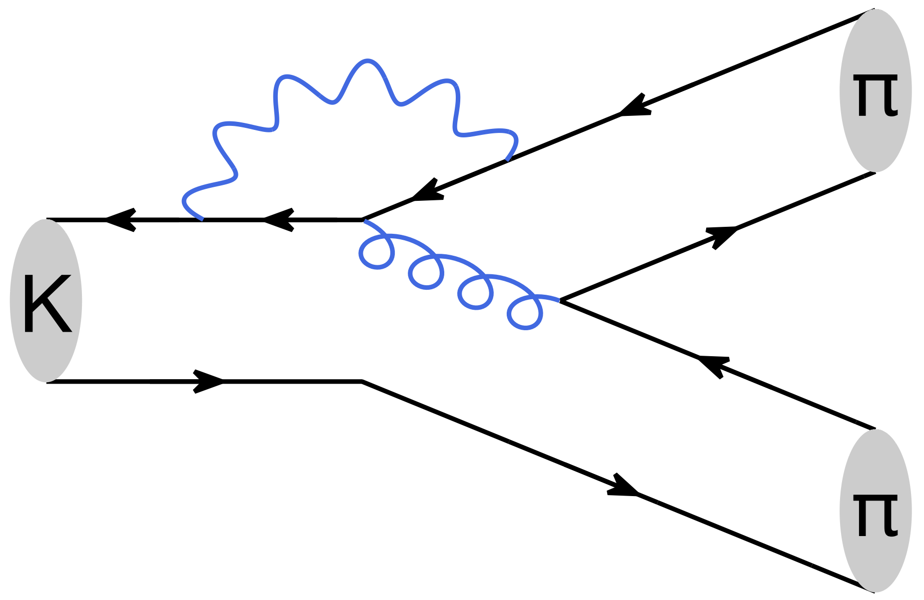
$|\pi\pi\rangle$

CP even

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \Big/ \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = 1 - 6\mathrm{Re}(\varepsilon'/\varepsilon)$$

- $|\varepsilon| = 2.228(11) \times 10^{-3}$ from “odd” mixing b/w K^0 & $\overline{K}^0$
- ε' only found in decays
  - $\mathrm{Re}(\varepsilon'/\varepsilon)_{exp} = 1.66(23) \times 10^{-3}$ (KTeV & NA48)
  - Consistent with SM?





K → ππ Amplitude and ε′

ππ phase shifts at m_K

$$\varepsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right] \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

Lellouch-Lüscher finite volume correction — Renormalization matrix

$$A_I = F\frac{G}{\sqrt{2}}V_{us}^{*}V_{ud}\sum_{i,j}[z_i(\mu)+\tau y_i(\mu)]Z_{ij}(\mu)\langle(\pi\pi)_I|Q_j^{lat}|K\rangle$$

Wilson coefs. pQCD — LQCD (+pQCD) — LQCD

- Matrix elements $< (\pi\pi)_I \,|\, Q_i^{lat} \,|\, K >$ from 3pt correlation functions
- A_2 amplitude has been determined very precisely [PRD91,074502 (2015)]
- A_0 challenging — disconnected diagrams, power divergences

Realizing on-shell kinematics

- The lowest-energy ππ state with “2 stationary pions” in Euclidean rest frame
  - $E_{\pi\pi,0} \approx 2m_\pi \approx 270$ MeV → off-shell
  - need $| E_{\pi\pi} = m_K \approx 500 \text{ MeV} \rangle$ state

- Possible approaches

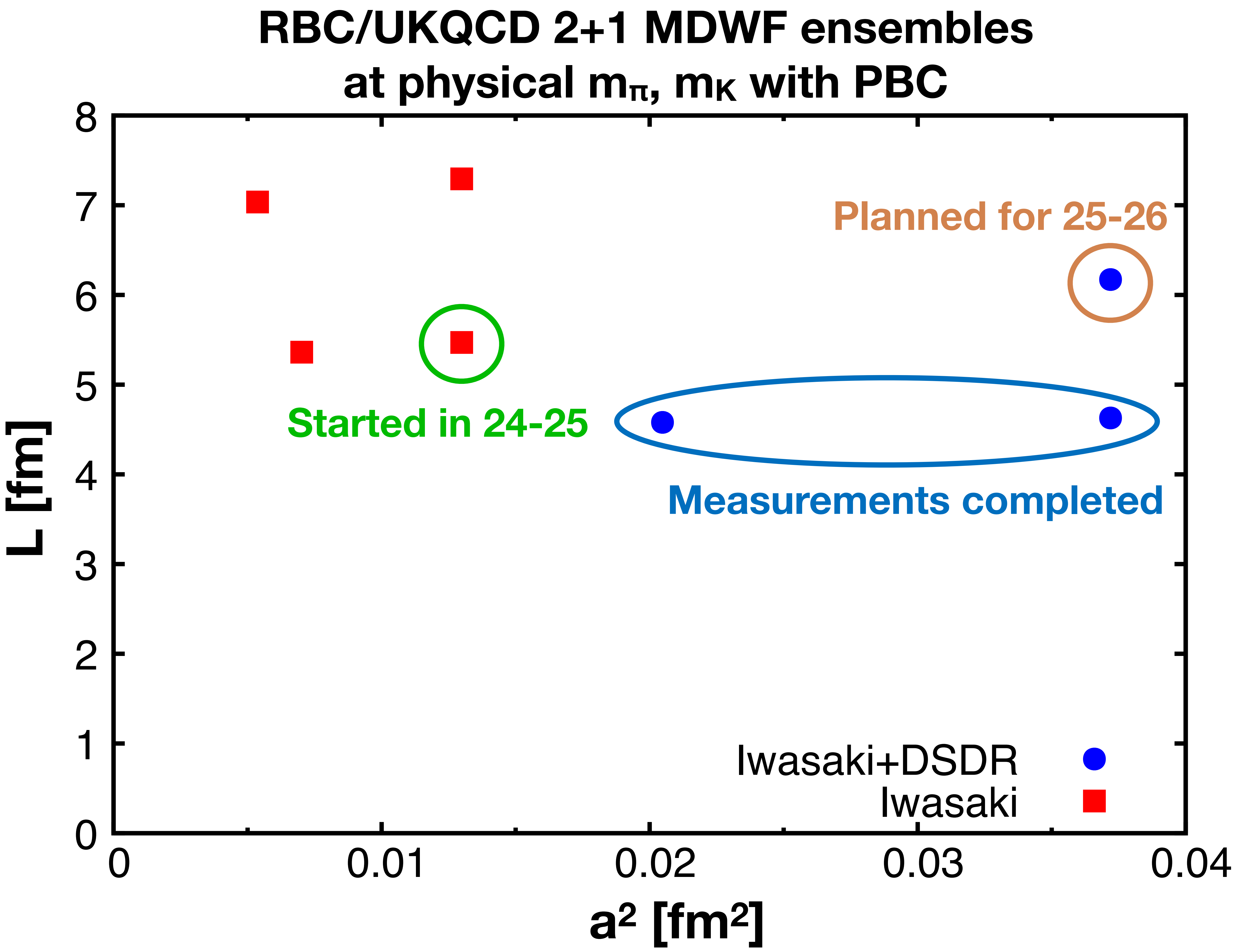
💡 Finite volume → two-pion spectrum not continuous

  - Manipulate boundary conditions → pions anti-periodic → must move → 500 MeV ground state possible [G-parity boundary conditions, C. Kelly]
  - Analyze lattice data taking multiple states into account with periodic boundary conditions **[THIS WORK]**
    - Many lattice ensembles with periodic boundary conditions already generated
    - suitable to introduce EM effects

Independent calculations with multiple approaches

Setup

- 2+1 MDWF
  - prevent unwanted operator mixings
- All-to-all propagator method
  - 2,000 low modes for ud (already generated)
  - spin-color-time diluted noise for high modes
- Sample AMA correction (exact calculation w 10x fewer confs)
  - CG 5x faster for sloppy
  - entire measurement 3x faster for sloppy



Extracting excited states

- GEVP

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$$

$C(t)$: N x N correlator matrix $C_{ab}(t) = \langle O_a(t)O_b(0)^\dagger \rangle$

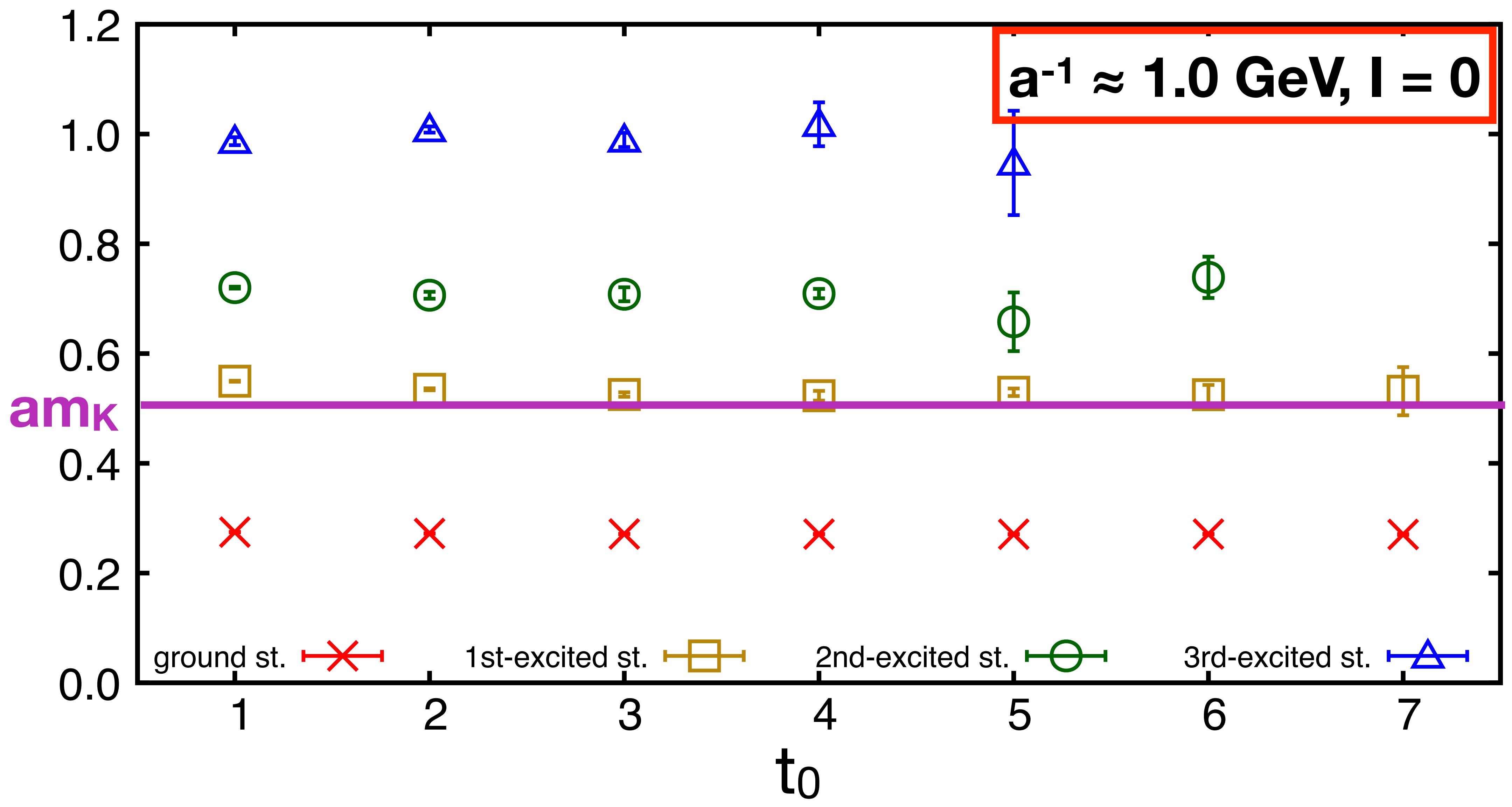
  - $O'_n = \sum_a v^*_{n,a} O_a$ couples mostly with the n-th state
  - $\lambda_n(t,t_0) = e^{-E_n(t-t_0)}$

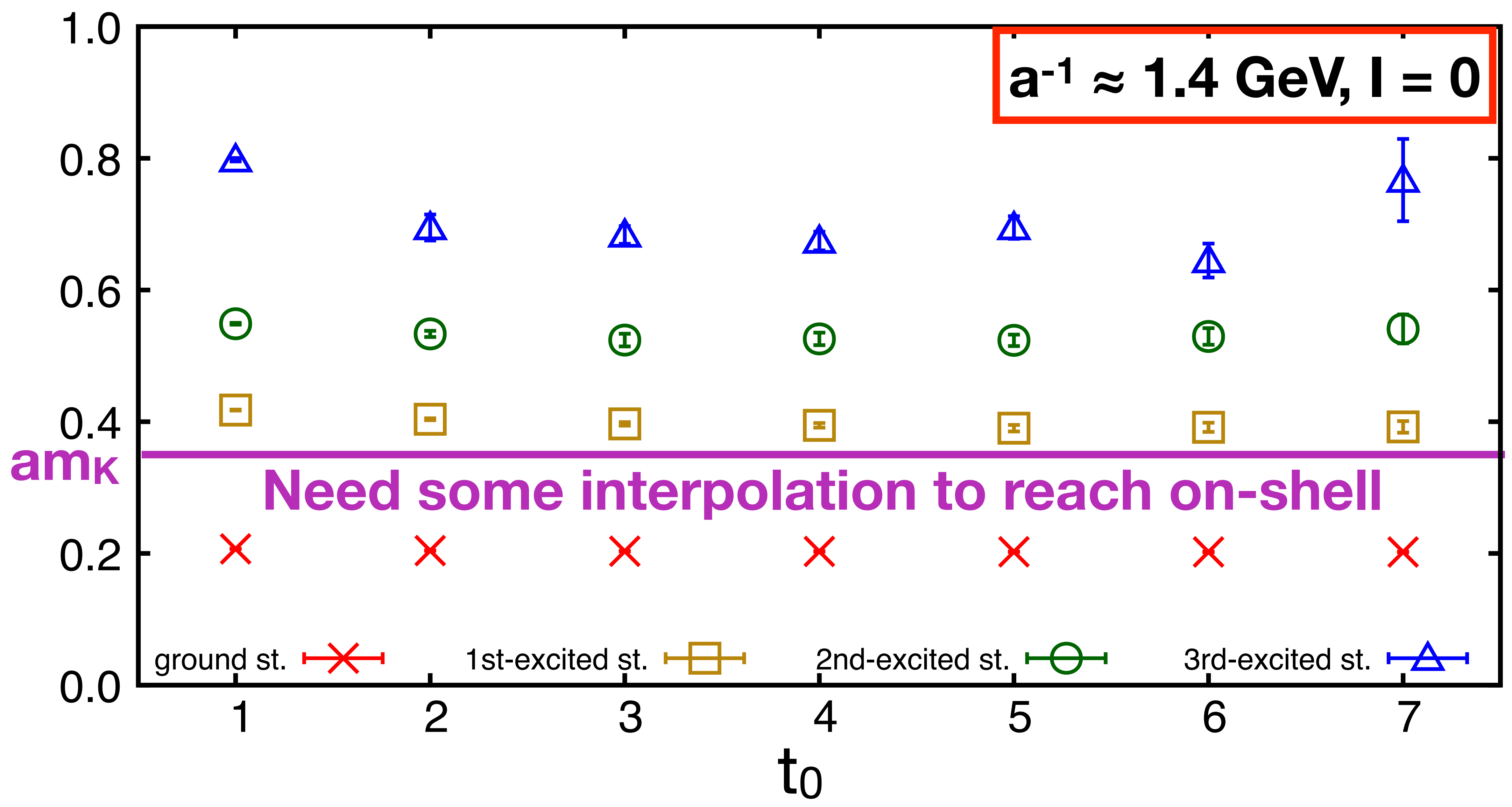
- Various ππ operators used in this work:
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$
  - $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$
  - $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$
  - $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$
  - σ ~ $\bar{u}u + \bar{d}d$ found important to control states around 500 MeV

The four ππ operators: **I = 2**

All five operators (ππ and σ): **I = 0**

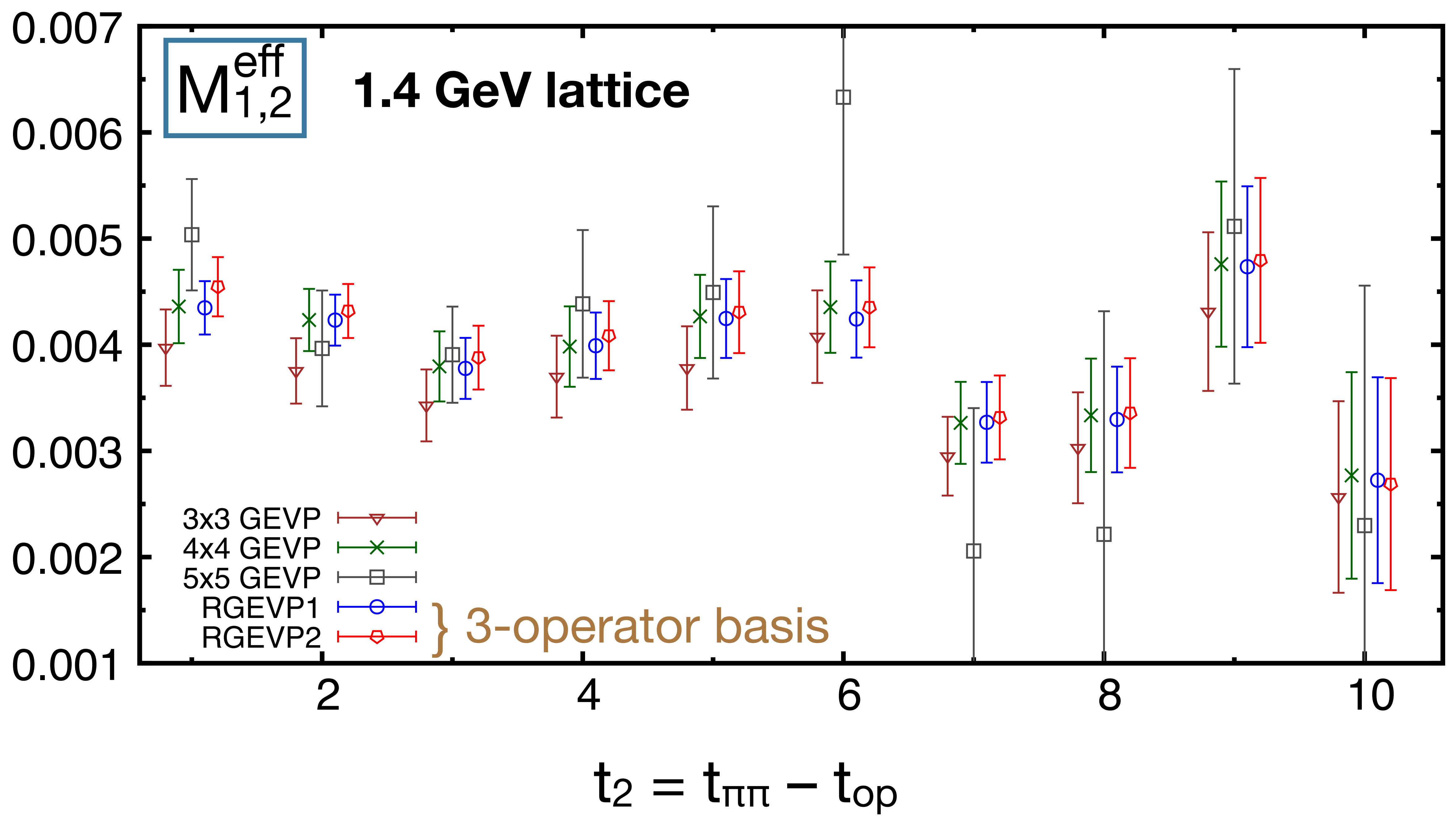
$aE_{\pi\pi}$ from ππ 2pt func & GEVP

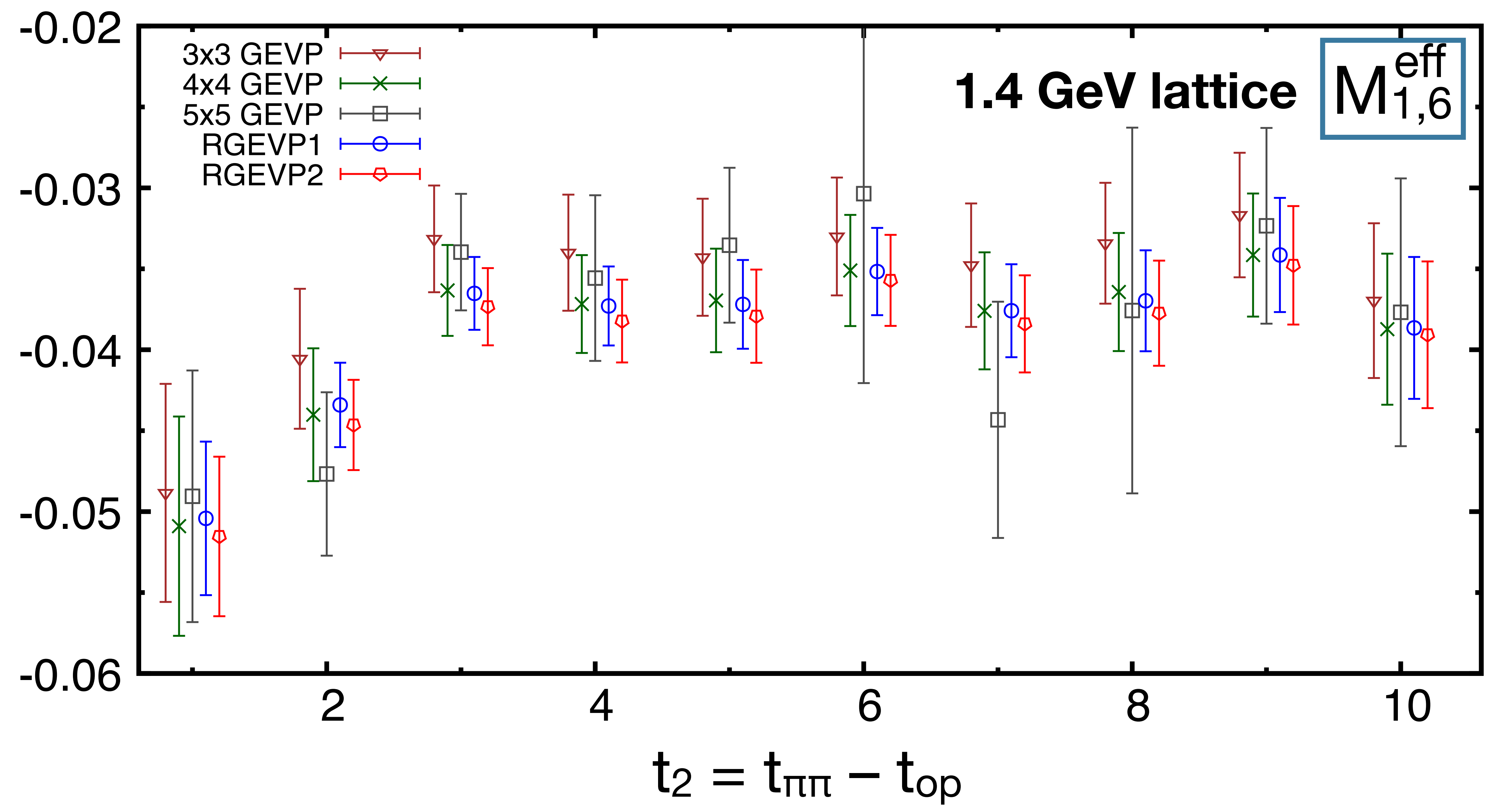




- Multiple states well resolved up to $E_{\pi\pi}$ ~ 1 GeV
- 1st-excited state energy close to m_K

Effective matrix elements





$$M^{eff}_{n,i}(t_2, t_1) = C^{(3)}_{n,i}(t_2, t_1) \left[\frac{e^{E^{\pi\pi}_n t_2} e^{E^K t_1}}{C^{\pi\pi}_n(t_2) C^K(t_1)} \right]^{1/2} \xrightarrow{\text{large } t_1 \text{ \& } t_2} M_{n,i}$$

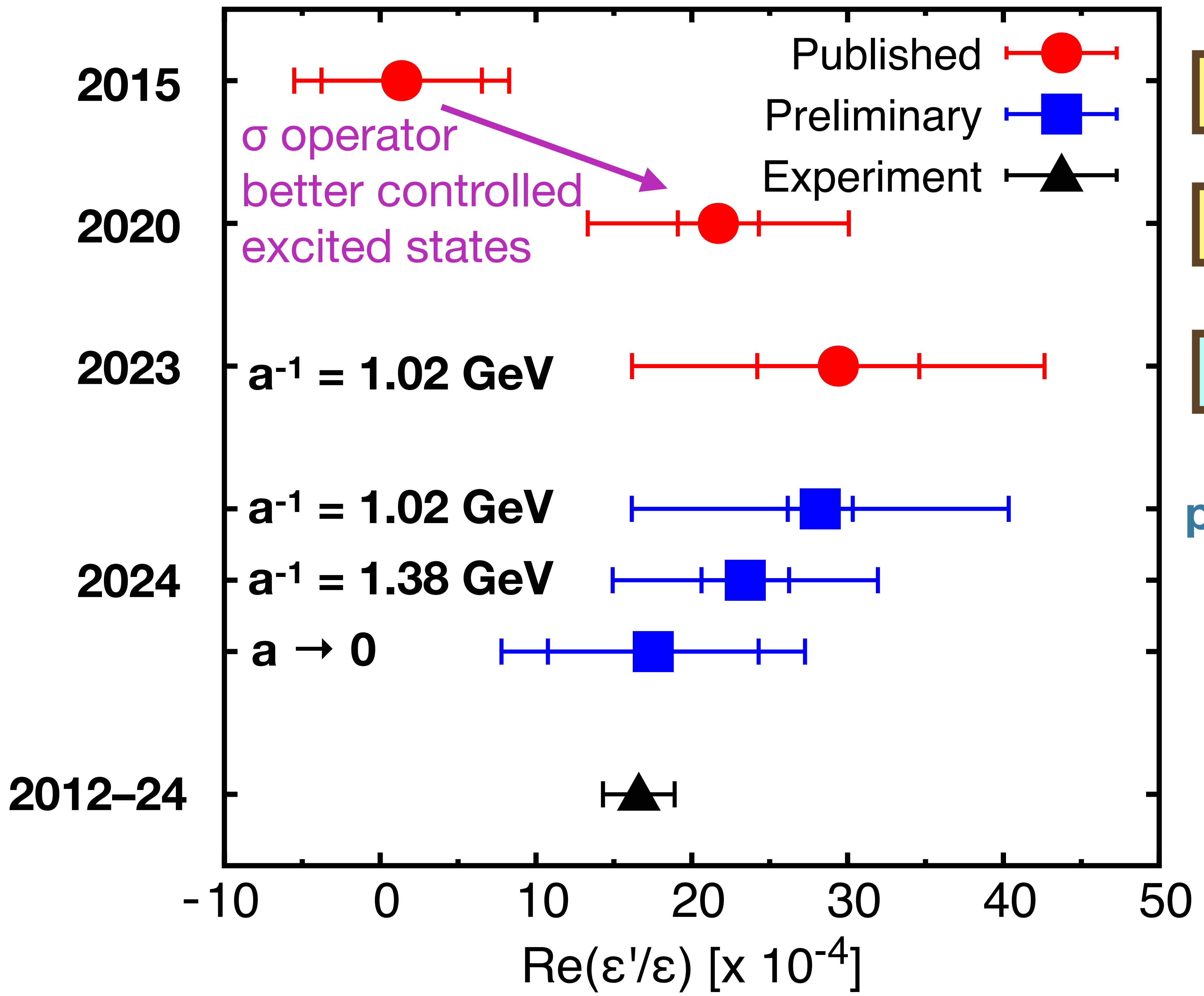
n: state index
i: operator index

Weighted average over large enough $t_1 = t_{op} - t_K$ taken

RBC/UKQCD achievements

Re(ε′/ε) x10[4]

RBC/UKQCD vs Experiment



PRL 115,212001

PRD 102,054509

G-parity Boundary Conditions (GPBC)

$a^{-1} \approx 1.38$ GeV

PRD 102,094517

preliminary results with multiple lattice spacings

Periodic Boundary Conditions (PBC)

$a^{-1} \approx 1.38$ GeV & 1.02 GeV

Plan for reducing systematic errors

- Systematic errors on Im A_0 (summarized in GPBC2020 paper)

Finite lattice spacing	12%
Wilson coefficients/charm-loop effects	12%
Lelloch-Lüscher FV correction	1.5%
Residual FV correction	7%
Parametric error	6%
Off-shellness	5%
Renormalization	4%
Missing G_1 operator	3%
TOTAL	21%

Finite lattice spacing → **Calculating on finer lattices towards a nice a → 0 limit**
****will be addressed by this project**

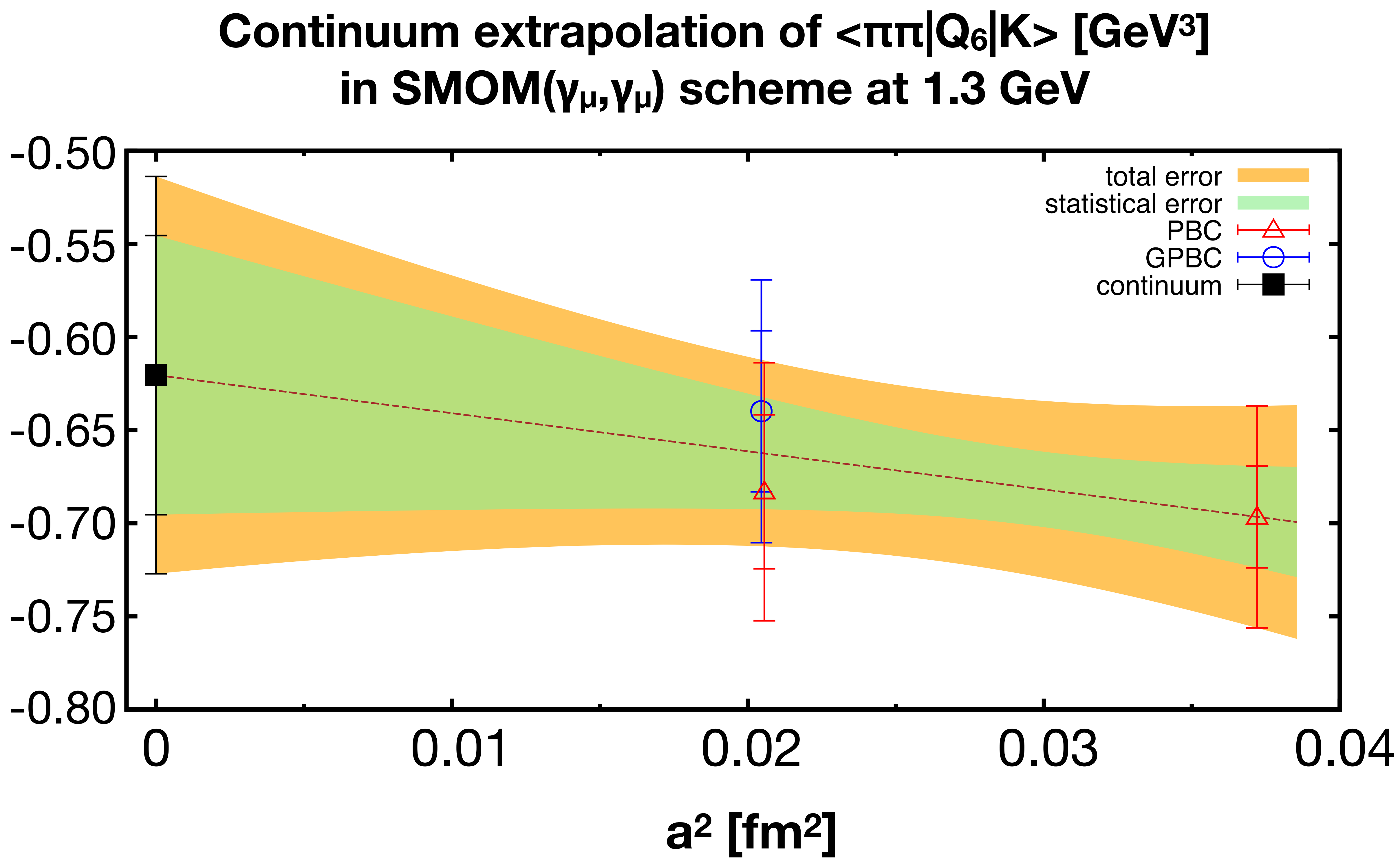
Wilson coefficients/charm-loop effects → **nonperturbative matching between 3 & 4-flavor theories to be performed**

- In addition
  - ε′ could be significantly affected by EM/IB effects (ΔI = 1/2 rule → ~20%)

Exploratory study underway
PBC appear necessary (GPBC violates charge conservation)

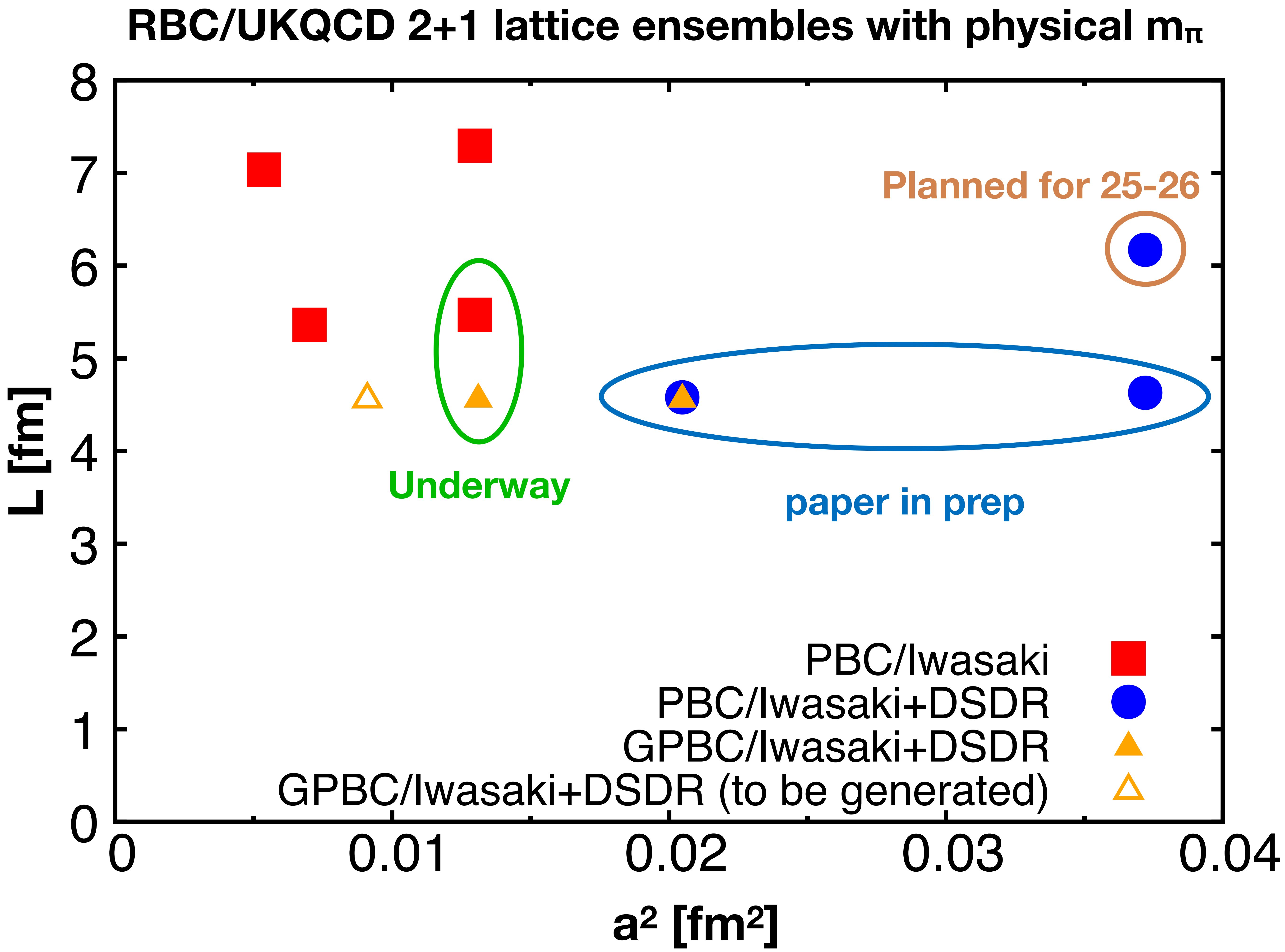
Continuum limit paper in prep

- Combining with GPBC result in 2020
  - Same fermion and gauge action
- $O(a^4)$ error potentially significant
  - careful estimation being discussed



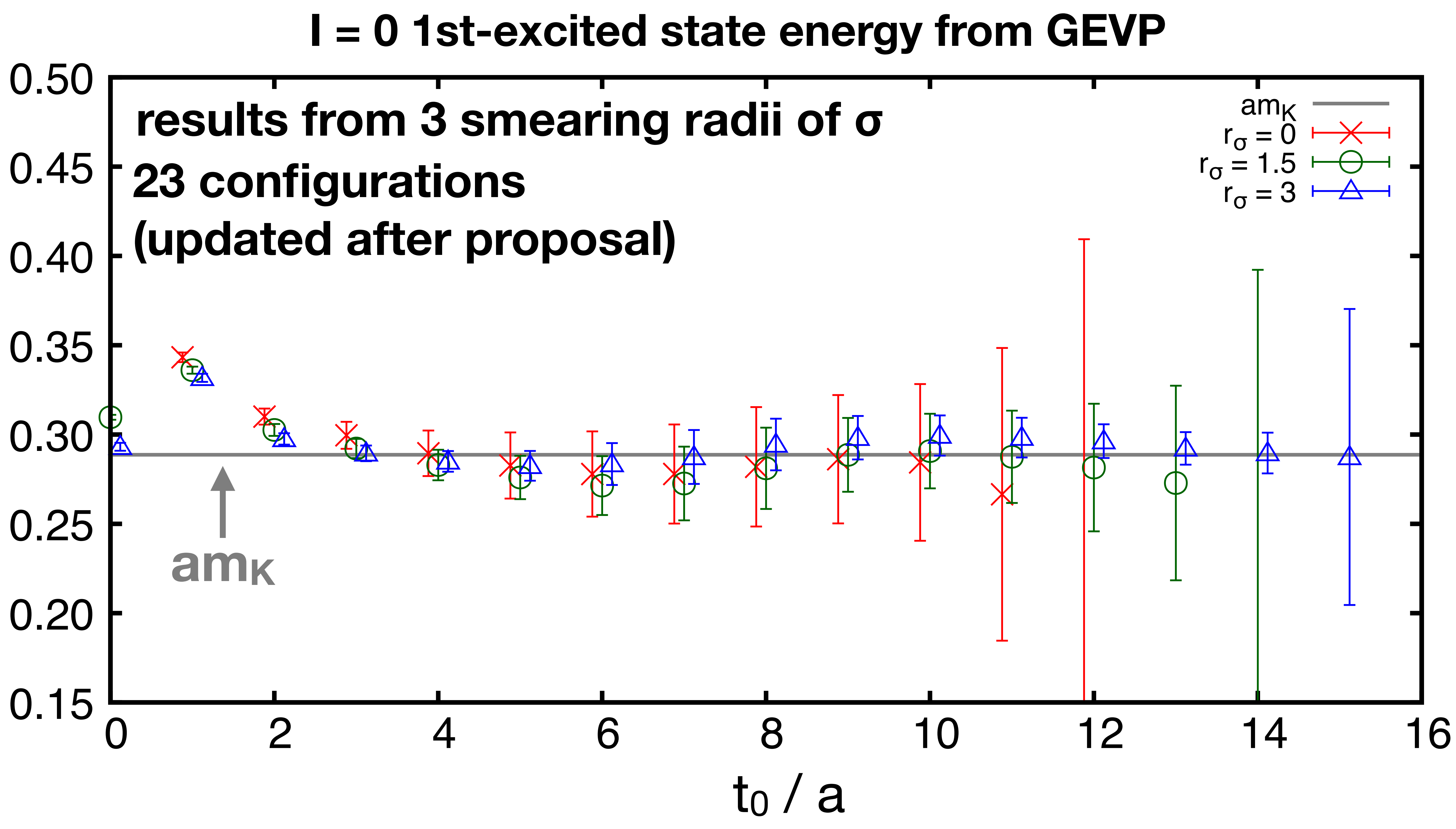
Plan for finer lattices

- Iwasaki+DSDR ensembles
  - PBC (●) & GPBC (▲)
  - GPBC continuing w a finer ensemble $a^{-1} \approx 1.7$ GeV (non-USQCD resource)
  - PBC w larger volume 32ID planned for 25-26 **(minor part of the proposal)**
- Iwasaki ensembles
  - Production going w PBC (■)
  - 48I ($a^{-1} \approx 1.7$ GeV, **major part of the proposal**)
  - Finer-lattice calculation may start soon with other resources



Status of 1.7 GeV lattice

- 1st-excited state energy very close to m_K
  - statistically consistent w current statistics
- Tuning parameters
  - r_σ: covariant smearing radius for σ operator
  - t_{sep} for $O_{\pi\pi}(t) = O_\pi(t)O_\pi(t+t_{sep})$
  - precision of $E_{\pi\pi}$ appears to depend on the parameters
  - more interesting to look at K→ππ matrix elements (will be done soon)



1st excited state energy coincides with mK
→ on-shell ME will be mostly obtained from this state

Summary

- Achievements with USQCD resources
  - Computation of $K \to \pi\pi$ amplitudes & ε' with multiple lattice spacings
  - Parameter tuning for 48I (48^3x96 at $a^{-1} \approx 1.7$ GeV) close to finish
- Goal for 25-26: Complete measurements on 48I and 32ID (large volume)
- Expected outcome
  - 48I: Significant improvement in the continuum limit combining with future results on finer lattices ($a^{-1} \approx 2.3$ GeV, …, other resources may be used)
  - 32ID: Better understanding of finite-volume effects & unphysical kinematics

Questions from SPC

1. **What is the source of discrepancy between the 2015 and 2020 GPBC results in Fig. 1?**

   – It was the lack of σ operator in 2015 that was causing significant underestimation of excited-state contamination.

2. **Will all sources of systematic uncertainties be accounted for in the ongoing PBC calculations? What is the estimated size of isospin-breaking effects and how do they compare with the target uncertainty on the value of (ε'/ε)?**

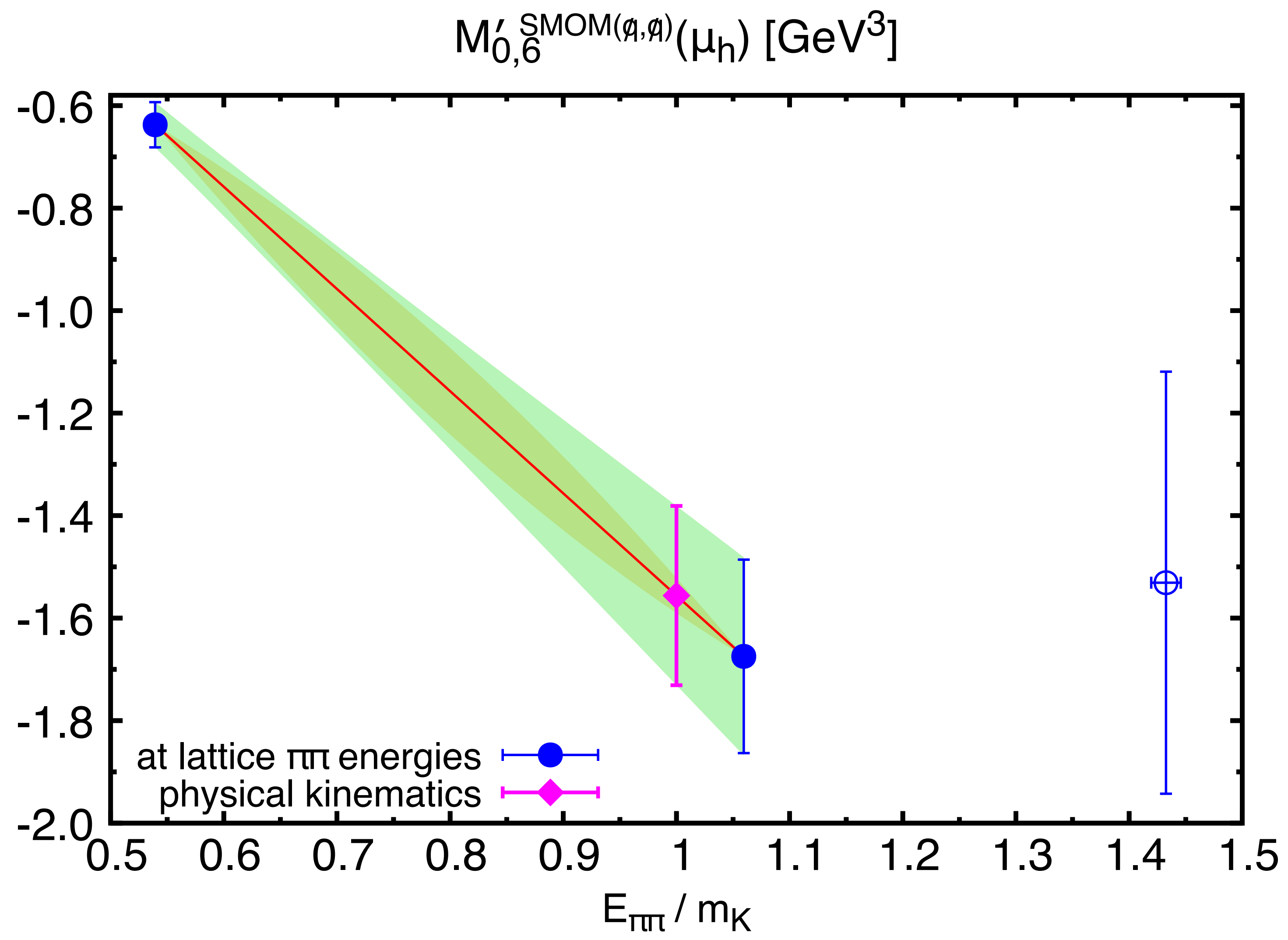
   – As discussed in slide 10, it would be the finite lattice spacing error that will be directly addressed through this project. While the uncertainty from EM/IB correction is expected larger than the experimental error (our ultimate target), there is a separate project underway to compute the correction. Another significant source of systematic error, perturbative truncation of Wilson coefficients, is also being addressed by another separate project of nonperturbative matching. We already have some results and are working hard to publish a paper on it this year.

Questions from SPC

3. **Since the calculations will involve a GVEP strategy and need a reconstruction of the physical amplitude using the Lellouch-Lüscher (LL) formalism, can you comment further on your physics-analysis strategy? How many kinematic points will you be able to access and how, and will they be sufficient to reach your uncertainty goal?**

– As shown in slide 7, we are able to resolve 4 states up to 1 GeV.

As I mentioned in the talk, we perform an interpolation using the ground and 1st-excited states. The figure on the right shows the interpolation. The 2nd excited state is above the inelastic threshold, where the LL formalism is not valid, and hence excluded from the interpolation. The guideline for the interpolation was based on the ChPT-based observation made in hep-lat/0208007 for the ΔI = 3/2 process (and we assume this is also the case for ΔI = 1/2). See also our paper in 2023 (PRD 102,094517) for the detail. Thanks to proximity of 1st excited state energy to m_K, **the systematic error is estimated substatistical and smaller than some other systematic errors.**



from PRD 102,094517

In order to better understand the unphysical kinematics effects, we will perform the calculation on larger volume (32ID) so more lattice data points in the figure can be obtained.

Again, slide 13 shows the 1st-excited state on 48I energy is more consistent with m_K so this kind of systematic error will be even smaller.

Questions from SPC

4. **You have requested some time on the AMD CPU cluster at BNL and have suggested even more usage of this cluster if needed. Can you confirm that you can organize more runs for this project at BNL compared to at JLAB? What would be the computing amount in AMD CPU hours needed to complete the proposed calculations entirely on BNL AMD CPU cluster?**

– We are mostly sure we can organize more runs for this project on the AMD cluster at BNL than our original request in the proposal. However, we would still like to receive non-zero allocation time at JLab so that we can run jobs when other users are not ready/able to run jobs (to avoid the significant undersubscription mentioned by Amitoj).

Our estimation based on a benchmark by co-PI, P. Boyle and C. Jung, is

2 Sky core hours ≈ 1 AMD core hour

for multi-node jobs. Whatever fraction is acceptable as long as our original request (24.1 M Sky-core-hours) is well respected and we receive non-zero allocation time for both 24s and AMD clusters. Please also make sure to increase our disk space (requesting +100 TB) at BNL so that we can smoothly perform production running.